February 14th, 1895, he sent me a French translation of the letter written in Magyar by John to his father from Temesvár, on November 3d, 1823, which fixes the date of John Bolyai's creation of the non-Euclidean geometry. I gave an English translation of this wonderful letter in the Introduction to the fourth edition of my "Science Absolute."

At last in July, 1896, in Budapest on my way to Maros-Vásárhely, I met and sojourned with Franz Schmidt himself, who long had been my correspondent. Many things he told me. For example, how first he came to know of John Bolyai. "My father," said he, "Anton Schmidt, was architect in Temesvár, 1817—1860, during which period he had often to build military structures. In later years he often told me of an officer of engineers, a Siebenburger (a Transylvanian, that is a Magyar from Erdély) with whom he always feared to come into contact. He related how this Magyar exhibited, to demonstrate the strength of his arm and the firmness of his Damascus sword, to each of his visitors, how he cut off with a single stroke stout iron nails which had been driven into the door post. That was our Bolyai János."

Franz Schmidt had visited Maros-Vásárhely in 1893 to find the grave of János, and arrange for the erection of a monument over it. He said: "John's grave was, even up to 1894, wholly unrecognizable, without any mark. Only his then still living nurse, Juliana Szöcs, who had attended upon him for eight years and who buried him, could show me the place." In the journal "Közérdek," published at Maros-Vásárhely, on November 11th, 1894, is the following note: "John Bolyai's grave for thirty-four years was without any mark, like to the ground. In 1893 Arehitect Franz Schmidt visited Maros-Vásárhely. To his indefatigable zeal we owe it, that the Mathematico-Physical Society of Budapest, by a subscription, erected to the author of the Appendix a pyramid of trachyte."

On my arrival at Maros-Vásáhely I had scarcely closed the door of my room at the "Transylvania" when in rushed a handsome old Magyar gentleman, threw his arms around me and kissed me! This was Professor Koncz József. The great chest containing John Bolyai's papers was opened for me. I was particularly struck with his writings on the Theory of Imaginaries, where he far surpassed Gauss, explaining the square root of minus one ($\sqrt{-1}$), the i of Gauss, as a new unit, for a new set of numbers qualitatively different from the numbers previously used, but no more imaginary than fractions or negative numbers. This is the view to which the world gradually attained unhelped by Bolgai's genius, for his work thereon, though sent to Leipzig to contest for a prize in 1838, had never been published. Long before seeing it I had suggested as a name for the new unit the word Neomon, giving neomic numbers in place of the unfortunately named imaginary numbers.

Even more was I struck with a manuscript treatise entitled "Raumlehre." Perhaps my enthusiasm was contagious. That year (1896) began a new epoch in the world's knowledge of Bolyai. That it was still Franz Schmidt will appear from the following excerpt which I translate from an article by Stäckel and

Engel, Math. Annalen, B. 49, pp. 149—206, 1897. "That we now find ourselves in more favorable position, is due before all to the persistence and devotion of architect Franz Schmidt of Budapest, who for thirty years continuously has worked to elucidate and expound the part of the two Bolyais in the history of the non-Euclidean geometry.

In December, 1896, the Royal Society of Sciences of Goettingen graciously put at his disposal a copy of the Correspondence between Wolfgang Bolgai and Gauss, and in concert with him one of us published in the Goettingen Nachrichten a selection of the mathematical part of this Correspondence.

But we owe still more to Architect Schmidt. In collaboration with his son, Prof. Dr. Martin Schmidt of Pressburg, he has subjected the papers left by the two Bolyais to a new inspection and has sent to us a series of communications about the results. But especially has he made accessible to us certain writings hitherto unknown, written in the Magyar language."

These Magyar documents are those into possession of which I had come years before, for a wonder independently of Franz Schmidt, and had translated, sending in August, 1895, a translation to Maros-Vásárhely to Professor Koncz József for him to review and annotate, This he did. But though I treasure his nine folio pages of annotation, I could not make up my mind to publish my translation simply because it was almost wholly about the father, Bolyai Farkas, while it is the son, Bolyai János, upon whom the world's interest should be centered. I wrote this to Professor Koncz in 1895 and begged for documents about János, for a picture of János, for any notes or remarks or anything pertaining to the immortal János, the most perfect case of genius in the world's history. On September 28th, 1895, he made a most precious and splendid response, as a few sentences from his letter will show. He says :

"I have the honor to send you : 1. Biographic data about Wolfgang Bolyai, written by John Bolyai. N. B. The lines underlined in red are not "conformes à la verité."

2. Biographic data on John Bolyai written by Coleman Szily, first Secretary of the Hungarian Academy of Sciences, from the notes of Gregory Bolyai.

3. The will of John Bolyai and his signatures at different epochs.

4. The photograph of Wolfgang Bolyai, taken on his death bed. N. B. There does not exist any portrait of John Bolyai.

5. An extract from the studies of John Bolyai in 1818.

I have not been able to procure any details about the duels of John Bolyai."

By the munificence of the Hungarian Academy of Science, Franz Schmidt was able in 1899 in conjunction with Paul Stäckel to publish the entire Bolyai-Gauss correspondence in a beautiful quarto of 208 pages. He had in 1897 at his own expense issued the first translation of the Science Absolute in Magyar, fulfilling thereby, as he says, a wish cherished for thirty years.

At the Bolyai János celebration next December, entwined in honor with the genius of the master must be the devotion of Franz Schmidt.

KINETIC DERIVATION OF TANGENT EQUATION.

By A. LATHAM BAKER, University of Rochester, Rochester, N. Y.

The following investigation is, I am afraid, more interesting than valuable, though it gives a very simple way of getting the equation of a tangent. But there is evidently an underlying principle which I have not succeeded in getting at, which would, if found, undoubtedly be valuable.

Since a curve can be considered as generated by the motion of a point, we can consider this point as the tracing point of a mechanism so adjusted as to trace the proper curve. This tracing point can be imagined to be the common point of various sets of x and y arms, each set composed of a certain number of x points, or of y points.

Now if, for example, the mechanism be set for the curve represented by the equation $ax^2+by^2=c$, and we stop the action of an x and a y point, leaving an x and a y point current, the new curve will be a straight line in *continuation* of the old motion. That is, the straight line will be a tangent line.

The algebraic equivalent of this would be found by writing

$$axx_1+byy_1=c$$

which would consequently be the *equation of the tangent line*, found by stopping one pair of coördinates (indicated by the subscripts) and letting the remaining pair remain current.

In the same way, from the algebraic mechanism

$$ax^m+by^m=c$$

we get the equation for the tangent line

$$ax_1{}^{m-1}x+by_1{}^{m-1}y=c.$$

Extending this thought to more complicated forms, we have the following divisions of algebraic mechanisms, using the word *simple* to indicate sets o terms containing only x's or only y's, and *compound* for terms containing both x's and y's.

A. TERMS SIMPLE AND COMPOSED OF THE SAME NUMBER OF TRACING POINTS, that is of the same degree, as in the examples already given, $ax^2+by^2=c$. In these we adopt a *current pair* of coördinates and make all the others fixed, viz., $ax_1x+by_1y=c$.

Example. $x+3y=a$. To have a fixed point (point of tangency) we must have two mechanisms compounded, viz.,

$$x+3y+x+3y=2a,$$

whence, leaving one x and one y current, we get $x+3y+x_1+3y_1=2a$.

B. TERMS SIMPLE, BUT WITH AN UNEQUAL NUMBER OF TRACING POINTS, with the subdivisions as to variable terms.

B_1. TWO TERMS ONLY, example $y=x^2$.

B_2. MORE THAN TWO TERMS BUT EACH TERM BALANCED BY ITS MATE IN THE OTHER LETTER.

B_3. MORE THAN TWO TERMS BUT THE TERMS NOT BALANCED.

B_1. As in A, we make one pair of tracing points (coördinates) current, but here the potency of the tracing point is inversely proportional to the number required to constitute a term, and the number of terms from which a current y point must be taken to balance a current x point must be *directly* proportional to the number of points in the term (degree of the term). Thus in $y=x^3+b$, one y must be balanced by three x's. Hence, taking three mechanisms together, we get

$$y+2y=3x^3+3b.$$

Now having current one y and three x's, we get $y+2y_1=3xx_1^2+3b$.

Similarly in $y^2=x^3$, two y's must be balanced by three x's and we must take $2y^2+y^2=3x^3$.

Leaving current two y's and three x's, we get $2yy_1+y_1^2=3xx_1^2$.

Again in $y^2=4px$, one x balances two y's, hence $2y^2=4px+4px$. Leaving current one x and two y's we get $2yy_1=4px+4px_1$.

B_2. Here we put each set of homogeneous terms equal to a summand of the absolute term, operate on these equations by $A-B_1$, and add the results.

Example. $y+y^2=x+x^2+c$.

$$\begin{array}{ll} y-x=p & y+y_1-(x+x_1)=2p \\ y^2-x^2=q & 2yy_1-2xx_1=2q \end{array}$$

whence $y+y_1+2yy_1=x+x_1+2xx_1+2c$.

B_3. Here we take the sum of the y's (or x's) and break it up into the sum of term of degree similar to the original, each summand being put equal to a term of the x's (or y's), and apply $A-B_2$. Or take the sets of homogeneous terms and place them equal to summands of the constant term; operate on these summands by $A-B_2$ and add the results.

Example. $y+y^2=x \qquad y=p \qquad y^2=q$

Whence $y+y_1=p+p_1$,

$2yy_1=q+q_1$. Whence $y+y_1+2yy_1=x+x_1$.

Example. $y+y^2=x^3$. $\qquad y=p^3, \qquad y+2y_1=3pp_1^2$

$y^2=q^3, \qquad 2yy_1+y_1^2=3qq_1^2$

Whence $y+2y_1+2yy_1+y_1^2=3xx_1^2$.

Example. $y=x^3-x^2+x$. $\qquad p=x^3, \qquad p+p_1=3xx_1^2-p_1$

$q=-x^2, \qquad q+q_1=-2xx_1$

$r=x, \qquad r+r_1=x+x_1$

Whence $y+y_1=3xx_1^2-2xx_1+x+x_1-x_1^3$.

Example. $y^2=x^3-x^2$. $p^2=x^3$, $2pp_1+p_1^2=3xx_1$
$q^2=-x^2$, $2qq_1=-2xx_1$
$q_1^2=-x_1^2$

Whence $2yy_1+y_1^2=3xx_1^2-2xx_1-x_1^3$. Here q_1^2 is added so that we can sum up to y's.

Example. $y+y^2=x^3$. $y=p$, $y+y_1=2p$
$y^2=q$, $2yy_1=2q$
$-x^3=r$, $x_1^3-3xx_1^2=2r$

Whence $y+y_1+2yy_1+x_1^3-3xx_1^2=0$.

Example. $y+y^2=x^3$. $y=p$, $y+2y_1=3p$
$y^2=q$, $y_1^2+2yy_1=3q$
$-x^3=r$, $-3xx_1^2=3r$

Whence $y+2y_1+y_1^2+2yy_1-3xx_1^2=0$.

Example. $y=x^3-x^2$. $p=x^3$, $p+p_1=3xx_1^2-x_1^3$
$q=-x^2$, $q+q_1=-2xx_1$

Whence $y+y_1=3xx_1^2-x_1^3-2xx_1$.

Example. $y+y_2=-x^2$. $y=-p^2$, $y+y_1=-2pp_1$
$y^2=-q^2$, $2yy_1=-2qq_1$

Whence $y+y_1+2yy_1=-2xx_1$.

Example. $y+y^2=x^3-x^2+x$. $p+p^2=x^3$, $p+2p_1+2pp_1+p_1^2=3xx_1^2$
$q+q^2=-x^2$, $q+q_1+2qq_1=-2xx_1$
$r+r^2=x$, $r+2rr_1+r_1=x+x_1$

Whence $y+2yy_1+y_1=3xx_1^2-2xx_1+x+x_1-p_1-p_1^2=3xx_1^2-2xx_1+x-x_1^3+x_1^2$.

Example. $y+y^2=x^3-x^2+x$. $y-x=p$, $y+y_1-x-x_1=2p$
$y^2+x^2=q$, $2(yy_1+xx_1)=2q$
$-x^3=r$, $-3xx_1^2-x_1^3=2r$

Whence $y+y_1-x-x_1+2yy_1+2xx_1=3xx_1^2-x_1^3$.

The value for $2r$ is found as follows. Suppose r variable say z, then $z+2z=-3x^3$, $z+2z_1=-3xx_1^2$, but once z is really constant this becomes $2z_1=2r=-3xx_1^2-x_1^3$.

C. Terms compound, but of equal degree.

C_1. The x's and y's balanced.

C_2. The x's and y's not balanced.

C_1. As in *A* make one pair current, taking each element from a different term.

Example. $xy=1$. $xy+xy=2$, so that a current y may be balanced by a current x and leave a fixed point. Whence $xy_1+x_1y=2$.

Example. $x^2y^2=c$. $x^2y^2+x^2y^2=2c$. Whence $xx_1y_1^2+yy_1x_1^2=2c$.

Example. $x^3y^3=c$. $x^3y^3+x^3y^3=2c$. Whence $x_1^3y_1^2y+xx_1^2y_1^3=3c$.

C_2. As in B_3, break up the absolute term into summands, each summand being equal to one of the class of terms, and proceed as in B_1, making one pair current, selecting its constituents from a number of terms inversely proportional to the weight of the elements selected, operating upon the terms as classified in $A-C_1$.

Example. $x^2y^3=c$. Here two x's must be balanced by three y's. Hence $2xx_1y_1^3+3yy_1^2x_1^2=5c$.

Example. $x^2y+xy^2=a$. $x^2y=a_1$, $2xx_1y_1+x_1^2y=3a_1$. $xy^2=a_2$, $2x_1y_1y+xy_1^2=3a_2$.

Whence $x(2x_1y_1+y_1^2)+y(2x_1y_1+x_1^2)=3a$.

D. Terms compound and not of the same degree.

D_1. x's and y's not balanced in each term.

D_2. x's and y's balanced in each term.

D_1. Decompose into summands as in B_3, C_1 and add the results.

Example. $x^2y+x^2y^3=a$. $x^2y=p$, $x_1^2y+2x_1y_1x=3p$ $x^2y^3=q$, $3x_1^2y_1^2+2x_1y_1^3x=5q$

Whence $x_1^2y+2x_1y_1x+3x_1^2y_1^2y+2x_1y_1^3x=3a+2q=3a+2x_1^2y_1^3$.

Example. $x^2y+x^3y=c$. $x^2y=p$, $x_1^2y+2xx_1y_1=3p$ $x^3y=q$, $x_1^3+3xx_1^2y_1=4q$

Whence $x_1^3y+3xx_1^2y_1+x_1^2y+2xx_1y_1=3c+x_1^3y_1$.

Example. $x^3y+xy=c$. $x^3y=p$, $x_1^3y+3xx_1^2y_1=4p$ $xy=q$, $x_1y+y_1x=2q$

Whence $x_1^3y+3xx_1^2y_1+x_1y+y_1x=2c+2p=2c+2x_1^3y_1$.

D_2. In D_1 owing to the unequal weight of x and y in the terms we are compelled by C_2 to take a number of terms proportional to the degree of the term before we can operate. In D_2 however, owing to the cancellations we must specifically designate this requirement or it will be overlooked. Hence, treat each term by itself and combine the results, each multiplied by the degree of its term.

Example. $xy+x^2y^2=a$. $x_1y+xy_1+2(x_1^2y_1y+xx_1y_1^2)=2p+4q=2a+2x_1^2y_1^2$.

Example. $x^4y^4+xy=a$. $x^4y^4=p$, $4(x_1^4y_1^3y+y_1^4x_1^3x)=8p$ $xy=q$, $x_1y+xy_1=2q$

Whence $4x_1^4y_1^3y+4xy_1^4x_1^3+x_1y+xy_1=8p+2q=2a+6x_1^4y_1^4$.

Example. $xy+x^3y^3=a$. $xy=p$, $x_1y+xy_1=2p$ $x^3y^3=q$, $3(xx_1^2y_1^3+x_1^3y_1^2y)=3.2q$

Whence $x_1y+y_1x+3xx_1^2y_1^3+3x_1^3y_1^2y=2p+6q=2a+4x_1y_1+4x_1^3y_1^3$.

Example. $x^2y^2+x^3y^3=a$. $2(x_1xy_1^2+x_1^2yy_1)+3(xx_1^2y_1^3+x_1^3yy_1^2)=4p+6q=4a+2x_1^3y_1^3$.

E. Composites of *A*, *B*, *C*, *D*.

Make each homogeneous set of terms equal to a summand of the absolute term. Operate on these by A−D, and add the results.

Example. $xy+x+y=a.$ $\quad xy=p,$ $\quad x_1y+xy_1=2p$

$x+y=q,$ $\quad x+x_1+y+y_1=2q$

Whence $x_1y+xy_1+x+y+x_1+y_1=2x_1y_1+2x_1+2y_1,$ and $x_1y+xy_1+x+y-x_1y_1=a.$

Example. $xy+x=1.$ $\quad xy=p,$ $\quad xy_1+x_1y=2p$

$x=q,$ $\quad x+x_1=2q$

Whence $x_1y+xy_1+x+x_1=2(p+q)=2.$

Example. $x^2+y^2+xy=1.$ $\quad x^2+y^2=p,$ $\quad 2(xx_1+yy_1)=2p$

$xy=q,$ $\quad xy_1+x_1y=2q$

Whence $xy_1+x_1y+2xx_1+2yy_1=2.$

Example. $x^2+xy=a.$ $\quad x^2=p,$ $\quad 2xx_1=2p$

$xy=q,$ $\quad xy_1+x_1y=2q$

Whence $x_1y+xy_1+2xx_1=2a.$

PROOF THAT FOR MAXIMUM CURRENT THE EXTERNAL AND INTERNAL RESISTANCES SHOULD BE EQUAL.

By JAMES S. STEVENS, Professor of Physics, University of Maine, Orono, Me.

If we have a cells to connect we may take m series with n cells in each series. Then $mn=a.$

By formula for Ohm's law,

$$C=\frac{nE}{\frac{nr}{m}+R}$$

where r and R are respectively the internal resistance of each cell and the total external resistance.

Dividing numerator and denominator by n we have

$$C=\frac{E}{\frac{r}{m}+\frac{R}{n}}.$$

For maximum current it is necessary to make $\frac{r}{m}+\frac{R}{n}$ a minimum.

The expression takes the following form:

$$\frac{R}{n}+\frac{r}{a/n}, \quad \frac{aR+rn^2}{an}.$$

Placing the first differential coefficient of this expression equal to zero we have

$$\frac{2an^2rdn-a^2Rdn-an^2rdn}{a^2n^2}=0.$$

From which

$$rn^2=aR, \quad n^2=\frac{aR}{r}.$$

Replacing the value of a/m for one factor in n^2,

$$n\frac{a}{m}=\frac{aR}{r}, \quad \frac{n}{m}=\frac{R}{r}, \quad R=\frac{nr}{m}.$$

Or the external resistance equals the total internal resistance. This is seen to be a minimum value for the expression differentiated since the value of the second differential coefficient is greater than zero for the positive value of n, —the only value it can have.

THE RADIUS OF THE TERRESTRIAL SPHEROID.

By F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

If there be nothing *new* under the sun, it may not be uninteresting to expand the *old*.

Represent the earth's equatorial radius by a, the geographical latitude by ϕ, and the geocentric latitude by ϕ'; then since $x^2/a^2+y^2/b^2=1$, we have $\tan\phi=-dx/dy$, and $\tan\phi'=y/x$. Also, since $b^2=a^2(1-e^2)$, we have

$$y^2=a^2(1-e^2)-(1-e^2)x^2 \text{ and } y/x=(1-e^2)\tan\phi.$$

$$\therefore x=\frac{a\cos\phi}{\sqrt{(1-e^2\sin^2\phi)}} \text{ and } y=\frac{a(1-e^2)\sin\phi}{\sqrt{(1-e^2\sin^2\phi)}} \ldots (1).$$

Now, the radius of the terrestrial spheroid for any latitude ϕ, is $\rho=\sqrt{(x^2+y^2)}$.

$$\therefore \rho=a\sqrt{\left(1-\frac{e^2(1-e^2)\sin^2\phi}{1-e^2\sin^2\phi}\right)}, =a\sqrt{[1-e^2(1-e^2)(\sin^2\phi+e^2\sin^4\phi)]}.$$

By assuming $e^2=1-f^2$ and $\sin^2\phi=\frac{1}{2}(1-\cos2\phi)$, Encke obtains the series

$$\log\rho=9.9992747+0.0007271\cos2\phi-0.0000018\cos4\phi,$$

in which the equatorial radius is unity.

Making $x=\rho\cos\phi'$ and $y=\rho\sin\phi'$, then the former equation of (1) may be written $e^2=\dfrac{\rho^2\cos^2\phi'-a^2\cos^2\phi}{\rho^2\sin^2\phi\cos^2\phi'}$; and by means of this value of e^2, the elimination of e^2 from the latter equation of (1) may be effected.

$$\therefore\ \rho^2\sin^2\phi'=\frac{(\rho^2\sin^2\phi\cos^2\phi'-\rho^2\cos^2\phi'+a^2\cos^2\phi)^2}{\rho^2\sin^2\phi\cos^2\phi\cos^2\phi'}.$$

$$\therefore\ \rho=a\sqrt{\left(\frac{\cos\phi}{\cos\phi'\cos(\phi-\phi')}\right)}\ldots.(a).$$

Formula (a) may be deduced in another way ; by assuming that

$$e\sin\phi=\sin\psi\ldots.(2),$$

we obtain

$$\rho\sin\phi'=a(1-e^2)\sin\phi\sec\psi\ \ldots(\alpha),\text{ and }\rho\cos\phi'=a\cos\phi\sec\psi\ldots.(\beta).$$

From (α) and (β) by easy deductions,

$$\rho\sin(\phi-\phi')=\tfrac{1}{2}ae^2\sin2\phi\sec\psi\ldots.(\alpha'),\text{ and }\rho\cos(\phi-\phi')=a\cos\psi\ldots.(\beta').$$

From (2) and (β') we have, respectively,

$$e=\sin\psi/\sin\phi\text{ and }\cos\psi=\rho\cos(\phi-\phi')/a\,;$$

and after transforming (α'), etc., we obtain

$$\rho\sin(\phi-\phi'),=a\times\frac{\cos\phi}{\sin\phi}\times\frac{1-\cos^2\psi}{\cos\psi},=a\times\frac{\cos\phi}{\sin\phi}\times\frac{a^2-\rho^2\cos^2(\phi-\phi')}{a\rho\cos(\phi-\phi')}.$$

$$\therefore\ \rho^2=\frac{a^2\cos\phi}{\sin(\phi-\phi')\cos(\phi-\phi')\sin\phi+\cos^2(\phi-\phi')\cos\phi}.$$

Expanding this denominator, combining terms, etc., we have (a) by a second method of reduction.

In order to obtain formula (a) by a third method, we remember that $a^2/b^2=\tan\phi/\tan\phi'$ and that $x^2+(a^2/b^2)y^2=a^2$; or after obvious transformations,

$$\rho^2\cos^2\phi'+(a^2/b^2)\rho^2\sin^2\phi'=a^2,\text{ or }\left[\cos^2\phi'+\left(\frac{\sin\phi\times\cos\phi'}{\cos\phi\times\sin\phi'}\right)\sin^2\phi'\right]\rho^2=a^2,$$

from which formula (a) is readily deduced.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

140. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah Iowa.

$\frac{1}{7}=0.\dot{1}4285\dot{7}$; $\frac{1}{14}=0.0\dot{7}1428\dot{7}$; $\frac{1}{21}=0.00\dot{4}7619\dot{0}$. Notice that the sum of the figures in each period is equal to 27. This is not true with $\frac{1}{7}\frac{1}{2}$, $\frac{1}{7}\frac{1}{3}$. Is there any general law of which these are special cases, and if so, what is it?

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

When the *period* of a circulating decimal fraction consists of an *even* number of figures, the second half of that period can be found by subtracting each figure of the first half from the figure 9. In this case the sum of the figures constituting a period is always divisible by 9, because we have as many 9's as there are figures in a half-period. Should the half-period have 3 or 6 or 9 or 12, etc., figures, then the sum of the figures in the period will be divisible by 27, and only then. The period of $\frac{1}{7}\frac{1}{2}$ is only 8 and cannot be divisible by 27. The period of $\frac{1}{7}\frac{1}{3}$ consists of eight figures of which the first four are 0, 1, 3, 6, consequently the last four are 9, 8, 6, 3, and the sum of the figures $4\times 9=36$.

141. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pa.

If the alloy in a half-dollar be 1-13th of the mass, and the coin be worth a cent if it be all alloy, what should be the exact value of the coin if it be all pure silver?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.; P. S. BERG, Larimore, N. D.; J. SCHEFFER, A. M., Hagerstown, Md.; and HARVEY M. DAVIS, Brown University, Providence, R. I.

Suppose the half-dollar worth 50 cents.

Then since alloy is worth $\frac{1}{13}$ of a cent, $\frac{12}{13}$ of the mass=silver, is worth $49\frac{12}{13}$ cents.

$\frac{1}{13}$ silver is worth $\frac{1}{12}$ of $49\frac{12}{13}$.

$\frac{13}{13}$ silver is worth $\frac{13}{12}$ of $49\frac{12}{13}$ cents$=54\frac{1}{2}$ cents.

GEOMETRY.

149. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

Given a conic and two circumscribing triangles of the conic; prove that the six vertices of the triangles are con-conic.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

This is most neatly solved, perhaps, by the theory of projection.

Projecting two of the vertices of the first triangle into the circular points

at infinity, the third vertex is the focus of the parabola into which the given conic becomes by projection.

The projection of the second triangle is a circumscribed triangle to the parabola, the circle circumscribing which triangle passing through the focus of the parabola, and proving the theorem.

The reciprocal theorem is: Two triangles are inscribed in a conic; their six sides touch another conic.

150. Proposed by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Find the equation to a sphere cutting orthogonally four given spheres.

Solution by the PROPOSER.

Let $x^2+y^2+z^2+2Ax+By+Cz+D=0 \ldots .(1)$, be the sphere cutting orthogonally the spheres

$$x^2+y^2+z^2+2a_1x+2b_1y+2c_1z+d_1=0 \ldots .(2),$$
$$\Sigma x^2+\Sigma 2a_2x+d_2=0 \ldots .(3),$$
$$\Sigma x^2+\Sigma 2a_3x+d_3=0 \ldots .(4),$$
$$\Sigma x^2+\Sigma 2a_4x+d_4=0 \ldots .(5),$$

Now, two spheres cut each other orthogonally if their radii and the distance between their centers form a right triangle; this requires, for (1) and (2),

$$(A-a_1)^2+(B-b_1)^2+(C-c_1)^2=A^2+B^2+C^2-D+a_1^2+b_1^2+c_1^2-d_1,$$
$$\text{or, } 2Aa_1+2Bb_1+2Cc_1-D-d_1=0 \ldots .(6).$$

Similarly for the intersection of (1) and each of (3), (4) and (5),

$$2Aa_2+2Bb_2+2Cc_2-D-d_2=0 \ldots .(7),$$
$$2Aa_3+2Bb_3+2Cc_3-D-d_3=0 \ldots .(8),$$
$$2Aa_4+2Bb_4+2Cc_4-D-d_4=0 \ldots .(9),$$

(6), (7), (8), and (9) give

$$\triangle A=\begin{vmatrix} 2b_1, & 2c_1, & -1, & d_1 \\ 2b_2, & 2c_2, & -1, & d_2 \\ 2b_3, & 2c_3, & -1, & d_3 \\ 2b_4, & 2c_4, & -1, & d_4 \end{vmatrix} \ldots .(10),$$

and like values for B, C, and D, which in (1) gives the required equation.

Similar demonstrations were received from *J. W. YOUNG, G. B. M. ZERR*, and *LON C. WALKER*.

MECHANICS.

113. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

A rough uniform rod, length $2a$, is placed with a length $c(>a)$ projecting over the edge of the table. Prove that the rod will begin to slide over the edge when it has turned through an angle $\tan^{-1}\left[\frac{\mu a^2}{a^2+9(c-a)^2}\right]$. [From Loudon's *Rigid Dynamics.*]

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

Let ψ be the angle through which the rod has rotated at the end of time t from the beginning of motion, μ=the coefficient of friction, W=the weight of the rod, and R=the normal reaction between the table and rod.

Taking moments about the point of contact of the table and rod,

$$\frac{W}{g}[\tfrac{1}{3}a^2+(c-a)^2]\frac{d^2\varphi}{dt^2}=W(c-a)\cos\varphi \ldots\ldots(1);$$

and resolving parallel and perpendicular to to the rod,

$$\mu R=W\sin\varphi+\frac{W}{g}(c-a)\,\frac{d\varphi^2}{dt^2} \ldots\ldots(2),$$

$$R=W\cos\varphi-\frac{W}{g}(c-a)\,\frac{d^2\varphi}{dt^2} \ldots\ldots(3).$$

Multiplying (1) by $2\frac{d\varphi}{dt}$ and integrating, and noticing that when $\psi=0$, $\frac{d\varphi}{dt}=0$,

$$[\tfrac{1}{3}a^2+(c-a)^2]\frac{d\varphi^2}{dt^2}=2g(c-a)\sin\varphi \ldots\ldots(4).$$

Solving (1) and (4) for $\frac{d^2\varphi}{dt^2}$ and $\frac{d\varphi^2}{dt^2}$, and substituting in (2) and (3),

$$\mu R=W\sin\varphi\frac{a^2+9(c-a)^2}{a^2+3(c-a)^2} \ldots\ldots(5),$$

$$R=W\cos\varphi\frac{a^2}{a^2+3(c-a)^2} \ldots\ldots(6).$$

Dividing (6) by (5),

$$\tan\varphi=\frac{\mu a^2}{a^2+9(c-a)^2}\ldots.(7).$$

Also solved by *G. R. DEAN*, and *G. B. M. ZERR*.

114. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Prove that the *inclination* of a perfectly rough inclined plane must be $\theta=\sin^{-1}[e^2/(2-e^2)]$, in order that an ellipse of minimum eccentricity e may be capable of resting in equilibrium on the plane.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The center of the ellipse will be vertically over the point of support; also the vertical through the point of support and the parallel to the plane through the center will form conjugate diameters.

$\therefore$ The acute angle between these diameters $=\frac{1}{2}\pi-\theta$. Since the potential energy is greatest at a point bordering on motion, the major axis will bisect the angle $\frac{1}{2}\pi-\theta$; hence the major axis makes with the conjugate diameters angles $\frac{1}{4}\pi-\frac{1}{2}\theta$ and $\frac{1}{4}(3\pi)+\frac{1}{2}\theta$.

We have for conjugate diameters, then, this condition,

$$a^2\sin^2[\tfrac{1}{4}\pi-\tfrac{1}{2}\theta]\sin[\tfrac{1}{4}(3\pi)+\tfrac{1}{2}\theta]+b^2\cos[\tfrac{1}{4}\pi-\tfrac{1}{2}\theta]\cos[\tfrac{1}{4}(3\pi)+\tfrac{1}{2}\theta]=0,$$

$$\text{or } a^2\sin^2(\tfrac{1}{4}\pi-\tfrac{1}{2}\theta)-b^2\cos^2(\tfrac{1}{4}\pi-\tfrac{1}{2}\theta)=0.$$

$$\therefore \frac{a^2-b^2}{a^2}=e^2=\frac{2\sin\theta}{1+\sin\theta} \text{ or } \sin\theta=\frac{e^2}{2-e^2}.$$

$$\therefore \theta=\sin^{-1}[e^2/(2-e^2)].$$

115. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A vessel in the shape of a parallelopiped, filled with water, has in its horizontal bottom a rectangular opening, whose dimensions are a and b, which is shut up by a slider. Supposing this slider to be opened with a uniform motion in the direction of a. To find the depth of the water in the vessel after the time T at the moment when the slider has passed through the space a, a denoting the horizontal section of the water in the vessel.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let h=height of parallelopiped, K=area of bottom, x=depth of water at any time, y=the distance the slider has opened at any time.

Then average area of orifice $=b\int_0^a y\,dy/\int_0^a dy=\frac{1}{2}ab$.

$$\therefore t=-\frac{2K}{ab\sqrt{2g}}\int\frac{dx}{\sqrt{x}}=-\frac{4K\sqrt{x}}{ab\sqrt{2g}}+C.$$

Since $x=h$ when $t=0$, $C=\dfrac{4K\sqrt{h}}{ab\sqrt{2g}}$.

$\therefore t=T=\dfrac{4K}{ab\sqrt{2g}}(\sqrt{h}-\sqrt{x})$. $\therefore x=\left(\dfrac{4K\sqrt{h}-Tab\sqrt{2g}}{4K}\right)^2$.

AVERAGE AND PROBABILITY.

98. Proposed by REV. PREBENDARY WHITWORTH, A. M.

A has £m and B has £n. They play for points until one of them has lost all his money. If α and β be the respective chances that A and B win any point, the expectation of the number of points played will be

$$\frac{n\alpha^n(\alpha^m-\beta^m)-m\beta^m(\alpha^n+\beta^n)}{(\alpha-\beta)(\alpha^{m+n}-\beta^{m+n})}.$$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let A_m=A's chance of winning, B_n=B's chance of winning.

Then nA_m, mB_n=A's and B's expectation, respectively.

$\therefore$ Expectation of number of points played=E.

Then $E=(nA_m-mB_n)/(\alpha-\beta)$.

Let A_x=A's chance when he has x pounds and B has $m+n-x$ pounds.

$$\therefore A_x=\frac{\beta}{\alpha+\beta}A_{x-1}+\frac{\alpha}{\alpha+\beta}A_{x+1}.$$

$A_x-A_{x-1}=(\alpha/\beta)(A_{x+1}-A_x)$. Giving x successive values from 1 to x we get $A_1-A_0=(\alpha/\beta)(A_2-A_1)$, $A_2-A_1=(\alpha/\beta)(A_3-A_2)$, etc.

By continued multiplication we get $A_1-A_0=(\alpha/\beta)^{x-1}(A_x-A_{x-1})$ or $A_x-A_{x-1}=(\beta/\alpha)^{x-1}(A_1-A_0)$.

Give x successive values from 1 to x and add

$$A_x-A_0=(A_1-A_0)[1+\beta/\alpha+(\beta/\alpha)^2+\ldots\ldots+(\beta/\alpha)^{x-1}].$$

But $A_0=0$. $\therefore A_x=A_1[1-(\beta/\alpha)^x]/[1-(\beta/\alpha)]$.

$A_{m+n}=1$. $\therefore 1=A_1[\alpha^{m+n}-\beta^{m+n}]/[\alpha^{m+n-1}(\alpha-\beta)]$.

$\therefore A_1=[\alpha^{m+n-1}(\alpha-\beta)]/(\alpha^{m+n}-\beta^{m+n})$.

$\therefore A_x=[\alpha^{m+n-1}(\alpha^x-\beta^x)]/[\alpha^{x-1}(\alpha^{m+n}-\beta^{m+n})]$.

$\therefore A_m=[\alpha^n(\alpha^m-\beta^m)]/(\alpha^{m+n}-\beta^{m+n})$.

Similarly, $B_n=[\beta^m(\alpha^n-\beta^n)]/(\alpha^{m+n}-\beta^{m+n})$.

$$\therefore E=\frac{n\alpha^n(\alpha^m-\beta^m)-m\beta^m(\alpha^n-\beta^n)}{(\alpha-\beta)(\alpha^{m+n}-\beta^{m+n})}.$$

99. Proposed by E. B. SEITZ.

A point is taken at random in the surface of a given circle, and from it a line equal in length to the radius is drawn, so as to lie wholly in the surface of the circle. Find the chance that the line intersects in a given diameter. [No. 135, *The Analyst.*]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be the random point, and AB the given diameter.

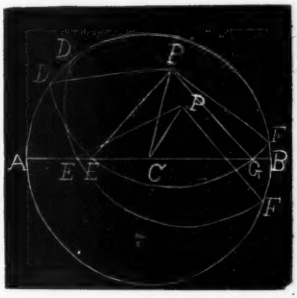
$CP=x$, $\angle PCB=\theta$. With P as center and radius$=r$, the radius of the given circle. Draw the arc DEF.

From $\theta=0$ to $\theta=\cos^{-1}\left(\frac{x}{2r}\right)$, F is below AB.

From $\theta=\cos^{-1}\left(\frac{x}{2r}\right)$ to $\theta=\frac{1}{2}\pi$, F is above AB.

Let $\cos^{-1}\left(\frac{x}{2r}\right)=\theta'$, p=the required chance. Then

$$p=\frac{\int_0^r\int_0^{\theta'} EF.xdxd\theta+\int_0^r\int_{\theta'}^{\frac{1}{2}\pi} EG.xdxd\theta}{\int_0^r\int_0^{\frac{1}{2}\pi} DF.xdxd\theta}.$$



Arc $EF=r[\theta-\sin^{-1}(\frac{x}{r}\sin\theta)+\cos^{-1}(\frac{x}{2r})]$, arc $DF=2r\cos^{-1}(\frac{x}{2r})$, arc $EG=r[\pi-2\sin^{-1}(\frac{x}{r}\sin\theta)]$.

$$\therefore\ p=\frac{\int_0^r\left(\int_0^{\theta'} EFd\theta+\int_{\theta'}^{\frac{1}{2}\pi} EGd\theta\right)xdx}{\int_0^r\int_0^{\frac{1}{2}\pi} 2r\cos^{-1}\left(\frac{x}{2r}\right)xdxd\theta}$$

$$=\frac{12}{\pi r^2(4\pi-3\sqrt{3})}\int_0^r\left(\int_0^{\theta'}[\theta-\sin^{-1}(\frac{x}{r}\sin\theta)+\cos^{-1}(\frac{x}{2r})]d\theta\right.$$

$$\left.+\int_{\theta'}^{\frac{1}{2}\pi}[\pi-2\sin^{-1}(\frac{x}{r}\sin\theta)]d\theta\right)xdx=\frac{12}{\pi r^2(4\pi-3\sqrt{3})}\left[\int_0^r\int_0^{\theta'}[\theta-\sin^{-1}(\frac{x}{r}\sin\theta)\right.$$

$$+\cos^{-1}(\frac{x}{2r})]xdxd\theta+\int_0^r\int_0^{\frac{1}{2}\pi}[\pi-2\sin^{-1}(\frac{x}{r}\sin\theta)]xdxd\theta$$

$$\left.-\int_0^r\int_0^{\theta'}[\pi-2\sin^{-1}(\frac{x}{r}\sin\theta)]xdxd\theta\right]$$

$$=\frac{12}{\pi r^2(4\pi-3\sqrt{3})}\left[\int_0^r\int_0^{\theta'}[\theta-\pi+\cos^{-1}(\frac{x}{2r})]xdxd\theta+\pi\int_0^r\int_0^{\frac{1}{2}\pi}xdxd\theta\right.$$

$$\left.+\int_0^{\frac{1}{2}\pi}\int_0^{2r\cos\theta}\sin^{-1}(\frac{x}{r}\sin\theta)d\theta xdx-2\int_0^{\frac{1}{2}\pi}\int_0^r\sin^{-1}(\frac{x}{r}\sin\theta)d\theta xdx\right]$$

$$= \frac{12}{\pi r^2(4\pi - 3\sqrt{3})}\left[\int_0^r \left(\tfrac{3}{2}(\cos^{-1}\frac{x}{2r})^2 - \pi\cos^{-1}\frac{x}{2r} + \tfrac{1}{2}\pi^2\right)x\,dx\right.$$

$$\left. + \int_0^{\frac{1}{3}\pi}(4r^2\theta\cos^2\theta + \tfrac{1}{2}r^2\cot\theta\cos 2\theta - \tfrac{1}{2}r^2\theta\operatorname{cosec}^2\theta)\,d\theta\right.$$

$$\left. - 2\int_0^{\frac{1}{3}\pi}(\tfrac{1}{2}r^2\theta + \tfrac{1}{4}r^2\cot\theta - \tfrac{1}{4}r^2\theta\operatorname{cosec}^2\theta)\,d\theta\right].$$

$$\therefore\ p = \frac{7\pi^2 + 8\pi\sqrt{3} - 54}{3\pi(4\pi - 3\sqrt{3})}.$$

NOTE.—This is problem 135 of *The Analyst*. Quite a discussion arose at the time of its first appearance, in which such eminent mathematicians of Prof. Benj. Pierce, of Harvard, took part. An incomplete solution by Professor Heaton was published. We believe Professor Zerr's solution is correct. ED. F.

100. Proposed by LON C. WALKER, Assistant in Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

Required the average distance between two points in opposite sides of a regular $2n$-gon.

Solution by J. SCHEFFER, A. M., Hagerstown, Md., and the PROPOSER.

From any point P at a distance x from the vertex A, in the side $AB = a$ of the regular polygon, draw PE perpendicular to the opposite side DC.

Let Q be any point in DC between D and E at the distance $= y$ from E. Then we have

$$PQ = \sqrt{a^2\cot^2\frac{\pi}{2n} + y^2}.$$

The average distance of all points in DE from the point P will be the same as the required average when the line PE is made to move parallel to itself the entire length of a side of the polygon.

The elements of AB and CD at the points P and Q are dx and dy, respectively. Therefore the required mean is

$$M = \frac{\int_0^a \int_0^x \sqrt{a^2\cot^2\frac{\pi}{2n} + y^2}\ dx\,dy}{\int_0^a \int_0^x dx\,dy}$$

$$= \frac{2}{a^2}\int_0^a \int_0^x \sqrt{a^2\cot^2\frac{\pi}{2n} + y^2}\ dx\,dy = \frac{1}{a^2}\int_0^a \left[x\sqrt{a^2\cot^2\frac{\pi}{2n} + x^2}\right.$$

$$\left. + a^2\cot^2\frac{\pi}{2n}\log\left(\frac{x + \sqrt{a^2\cot^2\frac{\pi}{2n} + x^2}}{a\cot\frac{\pi}{2n}}\right)\right]dx = \left[\tfrac{1}{3}\operatorname{cosec}\frac{\pi}{2n} - \tfrac{2}{3}\tan\frac{\pi}{4n}\cot^2\frac{\pi}{2n}\right.$$

$$+\cot^2\frac{\pi}{2n}\log\left(\frac{1+\sin\frac{\pi}{2n}}{\cos\frac{\pi}{2n}}\right)\Big].$$

COROLLARY. If $a=1$, $n=2$, then

$$M=\frac{2-\sqrt{2}}{3}+\log(1+\sqrt{2}).$$

Also solved by *G. B. M. ZERR*, and *HENRY HEATON*.

MISCELLANEOUS.

90. Proposed by DR. E. D. ROE, Jr., Syracuse University, Syracuse, N. Y.

I shot my rifle at different ranges and found the following table of elevations e, for the vernier peep sight, for the given distances s:

s	e
0	21.0
100	24.5
200	28.5
300	33.5
400	40.0
500	48.5

The distances are measured in yards. How shall a table of elevations be constructed, giving the arguments e, for every five yards up to 500 yards? Do not give the whole table, but explain the method, and illustrate by giving a computation, carrying the result to three places of decimals. An actual problem.

Solution by the PROPOSER.

The solution of this problem, which is doubtless somewhat arbitrary, was as follows: We have partly given and partly implied in the data, the following scheme:

s	0	100	200	300	400	500
e	21.0	24.5	28.5	33.5	40.0	48.5
Δe	3.5	4.0	5.0	6.5	8.5	
$\Delta^2 e$	0.5	1.0	1.5	2.0		
$\Delta^3 e$	0.5	0.5	0.5			
$\Delta^4 e$	0	0				

In this we notice that the fourth and third differences are constant, while the second are in arithmetical progression. Assuming that we have discovered the law, we may extend the second, third and fourth differences up to the column under 400, and thus have the complete data for interpolation by means of the method of finite differences. According to this method, if e_s denote the elevation for the distance s, we have,

$$e_{y\times 100+(x\cdot 20)\times 100}=e_{100y+5x}=e_{100y}+\frac{x}{20}\frac{\Delta e}{1\,!}100y+\frac{x}{20}\left(\frac{x}{20}-1\right)\frac{\Delta^2 e}{2\,!}100y$$

$$+\frac{x}{20}\left(\frac{x}{20}-1\right)\left(\frac{x}{20}-2\right)\frac{\Delta^3 e_{100y}}{2\,!},$$

where $y=0, 1, 2, 3, 4$.

$x=1, 2, \ldots 19$. As the complete solution, and giving the 95 values sought.

Example: It is required to find e_{425}. Here $y=4$, $x=5$, and $e_{400}=40$, $\Delta e_{400}=8.5$, $\Delta^2 e_{400}=2.5$, $\Delta^3 e_{400}=0.5$, $\Delta^4 e_{400}=0$. Hence

$$e_{425}=40+\frac{1}{4}\,8.5+\frac{1}{4}\left(\frac{1}{4}-1\right)\frac{2.5}{2\,!}+\frac{1}{4}\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-2\right)\frac{0.5}{3\,!}$$

$$=40+2.125-0.234+0.027=41.918.$$

Remark: With a table like this satisfactory results in hunting may be obtained. The table is contained in a small note book. But the style of hunting must be changed. Longer distances must be used, and the work resolves itself into judging distances, and variations in the wind, setting elevation and wind guage sights, accurate sighting and firm arm holding of the rifle.

Also solved by *G. B. M. ZERR*.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

144. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

A hired a house for one year for $300; at the end of four months he takes in M as a partner; and at the end of eight months he takes in P. At the end of the year what rent must each pay? [From Greenleaf's *National Arithmetic*, page 442.]

145. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

By discounting a note at 20% per annum, I get 22½% per annum interest; how long does the note run? [From Ray's *Higher Arithmetic*, page 405.]

*** Solutions of these problems should be sent to B. F. Finkel not later than July 10.

ALGEBRA.

136. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Solve $a^{x^2}b^{y^2}=c$.....(1), and $c^{x+y}=ab$.....(2).

137. Proposed by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

Solve, if possible, $a^x+b^x=c$.

138. Proposed by HARRY S. VANDIVER, Bala, Pa.

Show that the number of solutions in positive integers for x, y, and z of $x^3+2y^3+4z^3-6xyz=1$ is infinite.

*** Solutions of these problems should be sent to J. M. Colaw not later than July 10.

GEOMETRY.

166. Proposed by S. F. NORRIS, Professor of Astronomy and Mathematics, Baltimore City College, Baltimore, Md.

Two cities are 200 miles apart. To what height must a man ascend from one city in order that he may see the other, supposing the circumference of the earth to bo 25,000 miles? [From Wentworth's *New Plane and Solid Geometry*, page 381, No. 619.] Required solution by Geometry.

167. Proposed by JOHN J. QUINN, Professor of Mathematics, High School, Warren, Pa.

If at the vertex of an isosceles triangle one of whose basal vertices is pivoted and the other free to move in a straight line, a rhombus be pivoted with sides parallel to the sides of the triangle, the locus of every point on the rhombus except the one which is its intersection with the fixed side of the triangle is an ellipse.

168. Proposed by MISS GUBELMAN, Student Southern Illinois State University, Carbondale, Ill.

To draw a perpendicular to one side of a triangle dividing it into two equivalent parts.

*** Solutions of these problems should be sent to B. F. Finkel not later than July 10.

CALCULUS.

130. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Solve the differential equation $x^x(\frac{dy}{dx}+y\log x)-a=0$.

131. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College Mechanicsburg, Pa.

Integrate $2/x$, with regard to $d[\sqrt{(1-x^2)}]$.

132. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

What expression derived from the *polar* equation of a curve is equivalent to the expression for dy/dx derived from the *Cartesian* equation of the same curve? Prove work with $\rho=2r\cos\theta$.

*** Solutions of these problems should be sent to J. M. Colaw not later than July 10.

MECHANICS.

122. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Prove that a pressure P applied uniformly to a solid in all directions will reduce its dimensions along three perpendicular axes in ratio $1:1+p-2q$, p being the elongation along one face and q the contraction along the other. [Barker's *Physics*.]

123. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, Gloucestershire, England.

Two equal uniform rods AB, BC are freely hinged at B; C rests on a rough horizontal plane, and A is attached to a point above it. When C is as far as possible from A for equilibrium, AB, BC make angles α, β, respectively, with the vertical. Find the coefficient of friction between the rod at C and the plane.

*** Solutions of these problems should be sent to B. F. Finkel not later than July 10.

AVERAGE AND PROBABILITY.

107. Proposed by L. C. WALKER, Assistant Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.

Two points are taken at random in the curved surface of a hemisphere. Show (1) that the average length of the straight therein is $\frac{32r}{9\pi}$; and (2) that the average length of the arc of a great circle, which joins them, is $\frac{4r}{\pi}$.

108. Proposed by A. H. HOLMES, Brunswick, Me.

Required the average area of the quadrilateral whose sides are a, b, c, and d.

109. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A cylinder pierces a sphere in such a manner that the cylinder is tangent internally to the projection of the sphere in the plane xy. Find (1) the average surface, (2) the average volume of the sphere included within the cylinder.

*** Solutions of these problems should be sent to B. F. Finkel not later than July 10.

MISCELLANEOUS.

108. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

To divide the arc of a cardioid into eight equal parts.

109. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Find the latitude of the place where the sun's centre remains above the horizon for a hundred successive days.

In problem 105, Miscellaneous, April Number, Vol. VIII, No. 4,

$$\frac{2x}{a}=\frac{c}{a}l^{y/a}+\frac{a}{c}l^{-y/a} \text{ should be } \frac{2x}{a}=\frac{c}{a}e^{y/a}+\frac{a}{c}e^{-y/a}.$$

*** Solutions of these problems should be sent to J. M. Colaw not later than July 10.

NOTES.

Mr. H. W. Kuhn, of Cornell University, has been appointed Professor of Mathematics in the Ohio State University at Columbus, Ohio.

Dr. George Bruce Halsted has been invited to make at the Denver meeting of the Association for the Advancement of Science, August 24-31, 1901, a supplementary report on Non-Euclidean Geometry.

Professor H. A. Rowland, one of America's most noted physicists, died at Baltimore April 16. Professor Rowland was for a number of years past, Director of the Physical Laboratory at Johns Hopkins University.

Dr. E. A. Engler, Professor of Mathematics in Washington University, St. Louis, Mo., has been elected President of the Worcester Polytechnic Institute, to succeed Dr. T. C. Mendenhall who resigned on account of ill health.

Dr. G. A. Miller has recently been appointed Secretary of Section A of the American Association for the Advancement of Science in place of Professor Lord who resigned. Papers for the program of the Denver meeting should therefore be sent to Dr. Miller, 115 Cook St., Ithaca, N. Y.

BOOKS AND PERIODICALS.

Elements of Astronomy. By Simon Newcomb, Ph. D., LL. D., late Professor of Mathematics and Astronomy, Johns Hopkins University, formerly Senior Professor of Mathematics, U. S. Navy, and Superintendent of the American Ephemeries and Nautical Almanac. 1877-97. 12mo, cloth, 240 pages. Price, $1.00. New York and Chicago: American Book Co.

The general scope and plan of the book may be seen from the titles of the chapters: I. Relation of the Earth to the Heavens; II. The Revolution of the Earth Round the Sun; III. of Time; IV. Observation and Measurement of the Heavens; V. Gravitation; VI. The Earth; VII. The Sun. This book is written by America's foremost astronomer and, therefore, so far as the matter of the work is concerned no criticism can be offered. Neither can any criticism be offered as to the method of presentation. The author has certainly succeed in making a text-book simple and lucid enough to be comprehended by any one who has mastered the elements of arithmetic and the most rudimental principles of geometry, and at the same time it is full and complete enough for more advanced readers.

B. F. F.

The Riddle of the Universe at the Close of the Nineteenth Century. By Ernst Haeckel, Ph. D., M. D., LL. D., Sc. D., and Professor at the University of Jena, author of "The History of Creation," "The Evolution of Man," etc., translated by Joseph McCabe. 8vo, cloth, 390 pages. Price, $1.50. New York and London: Harper & Bros.

The scope of the work may be inferred from the titles of the chapters: I. The Nature of the problem; II. Our Bodily Frame; III. Our Life; IV. Our Embryonic Develop-

ment; V. The History of Our Species; VI. The Nature of the Soul; VII. Psychic Gradations; VIII. The Embryology of the Soul; IX. The Philogony of the Soul; X. Consciousness; XI. The Immortality of the Soul; XII. The Law of Substance; XIII. The Evolution of the World; XIV. The Unity of Nature; XV. God and World; XVI. Knowledge ahd Belief; XVII. Science and Christianity; XVIII. Our Monistic Religion; XIX. Our Monistic Ethics; XX. Solution of the World's Problems; Conclusion.

While the author has not solved the great "Riddle" to the entire satisfaction of every one who reads this work, yet he has handled the difficult problem with which he has dealt in a masterly way. A solution of this problem to satisfy all shades of belief is impossible, for there is only one solution, but there is an infinitude of beliefs and opinions. To read the correct solution, should it ever be given, one must lay aside all shades of beliefs and opinions and follow with an unbiased mind, step by step, the argument set forth. The author has not, in the light of more recent scientific discovery and progress, seen fit to change his opinions in reference to these great problems as formulated and expressed a quarter of a century ago. B. F. F.

The Measurement of General Exchange-Value. By Correa Moylan Walsh. Large 8vo, cloth, 580 pages. Price, $3.00, New York and London: The Macmillan Co.

This work should be in the hands of every teacher of Political Economy. In dealing with his subject, the author does not use any more mathematics than may be fairly expected to be known by a liberally educated person. B. F. F.

A Short History of the Greeks from the Earliest Times to B. C. 146. By Evelyn S. Shuckburgh, M. A., Late Fellow of Emanuel College, Cambridge. Author of "A Translation of Polybius," etc. 8vo. Cloth, 388 pages. Price, $1.10. New York and London: The Macmillan Co.

In this book, the story of the early Greeks is told with unusual interest. In developing the story, the author has chosen those topics which illustrate the political life and intellectual activities of the Greeks wherever they lived. B. F. F.

American Journal of Mathematics. Edited by Frank Morley, with the cooperation of Simon Newcomb, A. Cohen, Charlotte A. Scott, and other mathematicians. Published under the auspices of Johns Hopkins University. Issued quarterly. Price, $5.00 per year. Single Number, $1.50.

Number 2, Vol. XXII, contains the following articles: The Cross-ratio Group of 120 Quadratic Cremona Transformation of the Plane, by H. E. Slaught; Memoir on the Algebra of Symbolic Logic, by A. N. Whitehead; On a Special Form of Annular Surfaces, by Virgil Snyder; On the Transitive Substitution Groups whose Order is a Power of a Prime Number, by G. A. Miller; Geometry on the Cubic Scroll of the Second Kind, by Frederick C. Ferry. B. F. F.

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THE AMERICAN MATHEMATICAL MONTHLY.

Entered at the Post-office at Springfield, Missouri, as second-class matter.

VOL. VIII. JUNE-JULY, 1901. NOS. 6-7.

BIOGRAPHY.

CHARLES HERMITE.

By DR. GEORGE BRUCE HALSTED, Austin, Texas.

The fourteenth of Janunry, 1901, should be marked with a black stone in the annals of mathematics. Then the eminent geometer, the incomparable man, the great Hermite, one of the glories most pure of France, was lost to science, and implacable death threw into mourning his family, his friends, and his admirers.

As mathematician of the first rank he leaves to the glory of his country and of all humanity a superb scientific monument erected in sixty years completely dedicated to "his dear *analyse*" (to use one of his phrases) and to preparing by the infusion of his genius placed at the service of teaching that galaxy of illustrious mathematicians who now so much adorn our sister nation. Like Sturm, he united in an extraordinary degree the qualities of professor who wins the love of his disciples to those of one who inculcates the love of science for science. Endowed, like his compatriots, Pascal and Clairaut, with singular precocity, we see him, yet a scholar of the Lyceum Louis le Grand, win the prize for mathematics with a noteworthy thesis, and shortly after, as student of the Polytechnic School, attract the attention of Jacobi with his first works and place himself as of right in the first rank among the analysts of Europe.

It is not our object to make a minute analysis of the works of the great

geometer, to which would be necessary time and competence that we lack: our aim is much more modest: we seek to render what is heart-felt homage to the man we have so deeply venerated and from whom we have received infinite proofs of benevolence during the fifteen or sixteen years that we have had the honor to possess his friendship in so many ways precious. It is not possible, speaking of Charles Hermite, to fail to say how in the higher analysis, in algebra and in the theory of numbers one encounters everywhere the footprints of his giant tread. How could we leave unmentioned his memoir on the exponential function, where in demonstrating the transcendence of the number e he opens the way which eleven years after conducted Lindemann to the demonstration of the analogous property of π, solving in negative form the celebrated problem which for two thousand years had in vain fatigued geometers?

Nor can we pass in silence the enormous contribution which Hermite brought to the *Theory of Forms:* his law of reciprocity, his admirable researches on associate covariants, his works on quintic forms, his memoir on the equation of the fifth degree, and his celebrated theorem having Sturm's as corallary.

The works of Charles Hermite in the theory of functions are a new revelation of his genius. His profound investigations on Abelian functions, their division and their transformations, as also those relative to elliptic functions, form a monument of glory erected to French science, disclosing the sagacity of the grand analyst in the facility with which are deduced from the most lofty analytic investigations, corollaries which unveil difficult properties of the theory of numbers.

Neither can we neglect to mention the work "*Sur quelques applications des fonctions elliptiques*" (1885), of which only the first part was published: in this are found the beautiful applications of these functions which conduct him to the general integral of the equation of Lamé on the equilibrium of temperature of a homogeneous ellipsoid, which leads the author, in two particular cases, to the study of the rotation of a solid body around a fixed point (when there do not exist accelerating forces), treated by Jacobi, and to the consideration of the conic pendulum. So far as we know, Hermite leaves two didactic works: his "*Cours de la faculté des sciences de Paris*" (1891), and his "*Note sur la théorie des fonctions elliptiques*" (168 pages) which serves as appendix to the *Cours de calcul differentiel et integral* of J. A. Serret (4th ed. 1894). We have from him also two brief but interesting notes on the invariants of binary forms of the 5th and 6th order in the French translation of Salmons Higher Algebra.

The French geometer had the good fortune not granted all great men to see recognized in his lifetime by the scientific world his extraordinary merit. The 24th of December, 1892, his sixtieth birthday, the friends, the disciples, the admirers of the great geometer assembled at the Sorbonne to present him the gold medal struck in his honor by international subscription. The illustrious artist, Chaplain, cut upon it the bust of the one commemorated and translated onto metal with admirable fidelity his venerable face, affable and frank, illuminated by the scintilla of genius. The Minister of Public Instruction, M. Ch.